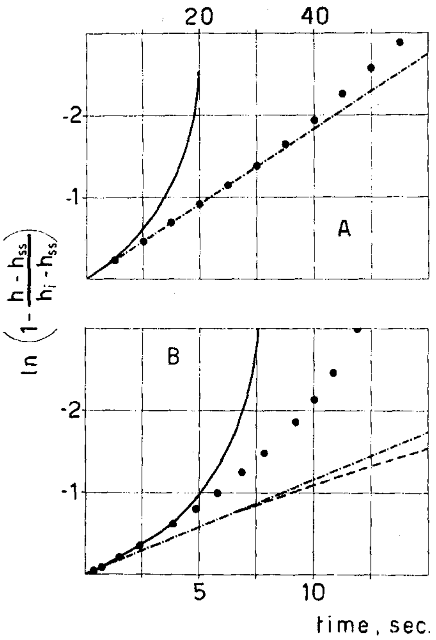

Fig. 3. Expansion of beds of glass beads. A—Reference 1, glass beads, 70 to 80 mesh; $\epsilon_{SS} \simeq 0.66$; $h_{SS} = 14.2$ cm.; $h_i = 26$ cm.; diameter of the column = 50 mm. B—Reference 3, glass beads, 20 to 25 mesh; $\epsilon_{SS} = 0.40$; $h_{SS} = 23.8$ cm.; $h_i = 51.8$ cm.; diameter of the column = 35 mm. ——— Piston flow. - - - - Complete mixing according to Equation (3). .... Linearized model for complete mixing according to Equation (5).

diagrams within theoretical curves for piston flow and complete mixing models.

NOTATION

h = bed height at time t, cm.
h_i = equilibrium bed height for superficial liquid velocity ϕ, cm.
h_{ss} = equilibrium bed height for superficial liquid velocity ϕ_{ss}, cm.
n = constant in Equation (1), dimensionless
t = time, sec.
T = theoretical time constant, sec.
U_s = terminal falling velocity of the particle, cm./sec.
ϵ = void fraction, dimensionless
ϵ_{ss} = equilibrium void degree for superficial liquid velocity ϕ_{ss}, dimensionless

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On a Generalized Expression for Prediction of Minimum Fluidization Velocity

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One of the important design variables that is encountered in the analysis and design of all fluid bed processes and which is required to be predicted with confidence is the minimum fluidization velocity. At the present time an approximate magnitude of this characteristic can be obtained on the basis of any one out of a few reliable empirical equations currently available (*2, 7, 8, 9, 11, 15*). An excellent review of the better approximation methods has been indicated in the recent work of Leva (*10*). Two approaches are evident in the analysis of the fluidization phenomenon. One of these is concerned with the valid hypothesis that at the point of initial bed expansion all equations pertaining to the behavior of fixed beds apply with equal rigor and that the pressure gradient is balanced by the net weight of the bed. Leva's analysis, which is considered to be one of the best, starts off by coupling the pressure drop equation for laminar flow through a bed of irregular particles with the above identity. The final equation for the prediction of minimum fluidization velocity is obtained on the basis of all available experimental data which establishes the variation of an empirical constant with Reynolds number, thus obviating the necessity to have prior values for bed voidage at minimum fluidization velocity and particle shape factor. Furthermore a correction factor, obtained through a chart (*10a*), is used as a multiplier in the original correlation when the particle Reynolds number exceeds the limit for laminar flow. Wilhelm and Kwauk (*15*) recommend a graphical procedure to be adopted on a plot connecting a dimensionless group $K_{\Delta P}$ (product of friction factor and the square of Reynolds number) and Reynolds number. The rest of the correlations are based on a second approach, namely dimensional analysis. The numerical constants of the final expressions are fixed with the help of experimental data. When a comparison is made of the predicted results from the different correlations for the same particle-fluid system, considerable deviation exists among them, and the extent of deviation appears to be a

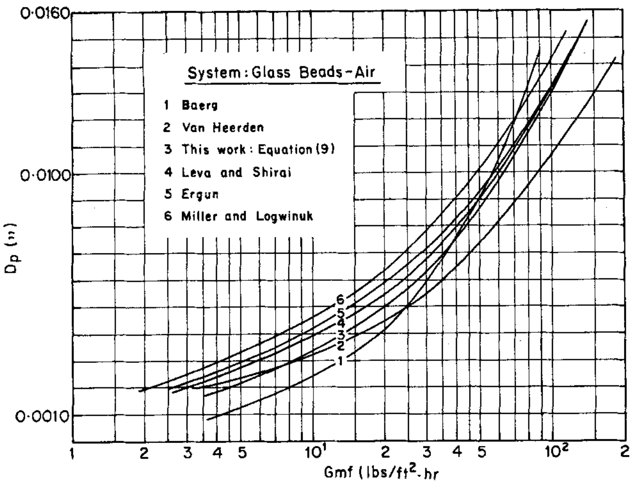

Fig. 1. Comparison of existing correlations for prediction of G_{mf}.

function of the particle diameter. This is apparent from Figure 1. However, while extreme values, high and low, are predicted by the equations of Miller (*11*) and Baerg (*2*), respectively, mean values are obtained through the equation of Leva (*9*) considered to be quite conservative. The present investigation represents an attempt to derive some generalized expressions that would directly predict minimum fluidization velocities in a range of particle Reynolds number not restricted to laminar region alone and without need to establish numerical constants by recourse to expermental data.

DERIVATION

The starting equation will be the generalized pressure drop correlation of Ergun (*6*) for fixed beds, duly corrected for particle shape factor:

$$\frac{\Delta P g_c}{L} = \frac{150(1-\epsilon)^2 \cdot \mu \cdot u}{\epsilon^3 \cdot \phi_s^2 \cdot D_p^2} + \frac{1.75(1-\epsilon) \cdot G \cdot u}{\epsilon^3 \cdot \phi_s \cdot D_p} \quad (1)$$

This theoretical equation has been tested to be valid over a wide and continuous range of Reynolds number covering the laminar, transition, and turbulent regions. At the point of initial bed expansion, the pressure drop across the bed may be equated as follows (*5*):

$$\frac{\Delta P_{mf}}{L} = (1-\epsilon_{mf})(\rho_s - \rho_f) \quad (2)$$

When one notes that $G = u \cdot \rho_f$ and that at the point of initial bed expansion $G = G_{mf}$, substitution of Equation (2) in (1) provides

$$(1-\epsilon_{mf})(\rho_s-\rho_f) \cdot g_c = \frac{150(1-\epsilon_{mf})^2 \cdot \mu \cdot G_{mf}}{\epsilon_{mf}^3 \cdot \rho_f \cdot \phi_s^2 \cdot D_p^2}$$

$$+ \frac{1.75(1-\epsilon_{mf})G_{mf}^2}{\epsilon_{mf}^3 \cdot \rho_f \cdot \phi_s \cdot D_p} \quad (3)$$

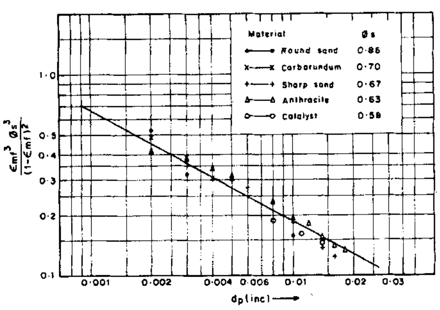

Fig. 2. Correlation of voidage shape factor function.

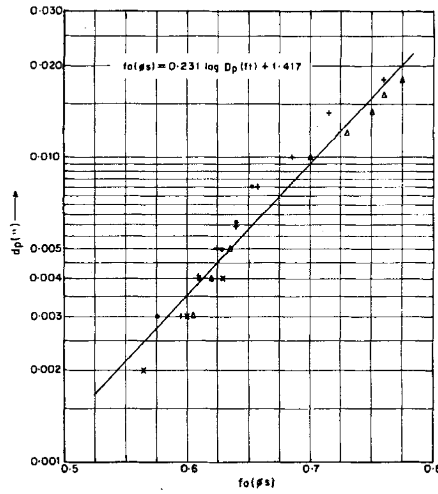

Fig. 3. Correlation of voidage shape factor function, *fo* (ϕs).

A similar expression, uncorrected for particle irregularities, has been indicated generally by Bennet and Myers (*3*). Actually Equation (3) is strictly applicable for only a differential depth of bed owing to the dependence of velocity on fluid expansion. For solid-liquid systems, however, the equation is rigidly valid. If the assumpton that an average velocity will be adequate for yielding results well within the limits of experimental accuracy is made, Equation (3) may be considered valid over the entire bed depth. Ergun (*5*) utilized a simplified form of Equation (3) for predicting the variation of bed porosity with bed expansion for short columns of fine low-density materials.

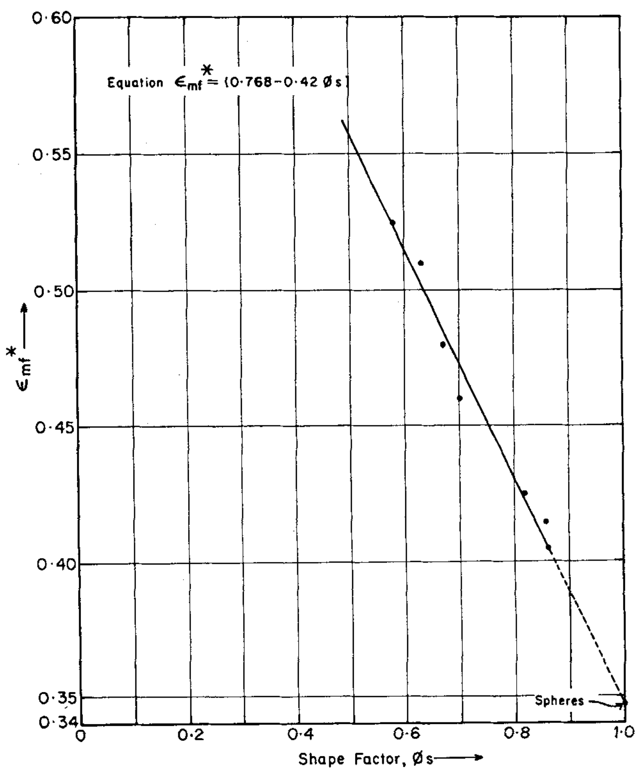

Fig. 4. Dependence of limiting values of ϵ_{mf} (for large particle diameters) on shape factor.

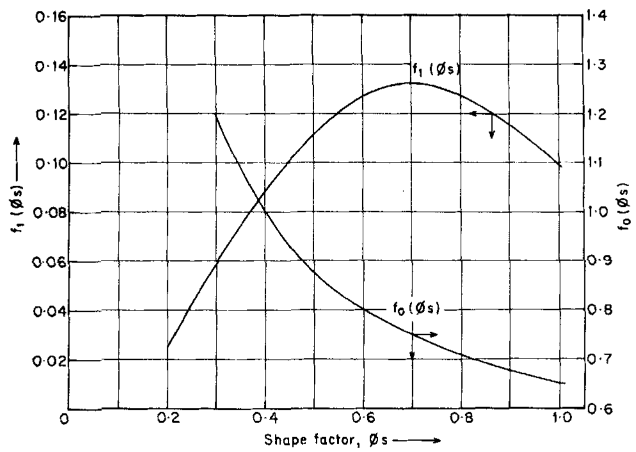

Fig. 5. Dependence of voidage shape factor functions on shape factor for large particle diameters.

For general applicability, if the original form is retained, a quadratic in G_{mf} may be obtained as follows:

$$G_{mf}^2 + \frac{85.8(1-\epsilon_{mf})\mu}{\phi_s \cdot D_p} G_{mf} - \frac{D_p \cdot \rho_f(\rho_s - \rho_f) g_c \cdot \phi_s \cdot \epsilon_{mf}^3}{1.75} = 0 \quad (4)$$

Extracting the real root of Equation (4) provides

$$G_{mf} = \frac{42.9(1-\epsilon_{mf}) \cdot \mu}{\phi_s \cdot D_p} \left\{ \left[1 + \frac{0.0056\, \epsilon_{mf}^3 \cdot \phi_s^3}{(1-\epsilon_{mf})^2} N_{Re_t} \right]^{1/2} - 1 \right\} \quad (5)$$

Equation (5) can directly predict the magnitude of minimum fluidization velocity provided the values of minimum fluidization voidage and particle shape factor are known. Unfortunately, these values are available for but a few systems. For a packed bed of uniform spheres, however, the minimum fluidization voidage may be assumed to be independent of particle diameter provided the wall effect is absent. An approximate value of 0.35 may be assumed for this parameter. The justification for assuming this value will be cited later. Therefore on substituting $\epsilon_{mf} = 0.35$ and $\phi_s = 1.0$ in Equation (5), an expression for the prediction of G_{mf} for a bed of uniform spheres in any flow regime may be given as

$$G_{mf} = \frac{27.83 \cdot \mu}{D_p} [(1 + 0.00057\, N_{Re_t})^{1/2} - 1] \quad (6)$$

For nonspherical particles, it is necessary to relate the shape factor-voidage functions, namely $\frac{(1-\epsilon_{mf})}{\phi_s}$ and $\frac{\epsilon_{mf}^3 \cdot \phi_s^3}{(1-\epsilon_{mf})^2}$ appearing in Equation (5), to one or more of the principal variables of the system based on available data. A clue to the appropriate choice of the variable lies in the presentation of classified ϵ_{mf} values as dependent on particle diameter and shape factor (*10b*). Shirai (9) reproduced these data by plotting the function $\epsilon_{mf}^3 \cdot \phi_s^2/(1-\epsilon_{mf})$ against particle diameter and obtained fairly consistent curves for different substances lying close together. Leva's analysis was restricted to the streamline region only, and the shape factor-voidage parameter was indirectly determined from published data on G_{mf} and then correlated against Reynolds number. The derived analytical function was substituted back on the original equation to provide the well-known expression for the prediction of G_{mf}. In the present case the following two expressions have been obtained based on suitable plots of available data which appear in Figures 2 and 3:

$$\frac{\epsilon_{mf}^3 \cdot \phi_s^3}{(1-\epsilon_{mf})^2} = 0.00379\, D_p^{-0.55} \quad (7)$$

$$\frac{(1-\epsilon_{mf})}{\phi_s} = 0.231 \log D_p + 1.417 \quad (8)$$

Equations (7) and (8) may be substituted in Equation (5) to give the following expression for the prediction of G_{mf} for irregular particles:

$$G_{mf} = \frac{42.9\, \mu}{D_p} (0.231 \log D_p + 1.417) [(1 + 2.12 \times 10^{-5} \cdot D_p^{-0.55} \cdot N_{Re_t})^{1/2} - 1] \quad (9)$$

The validity of Equations (6) and (9) is restricted to a size range of particles to which the original data pertain, namely 0.001 to 0.02 in. When a survey is made of all the published data, it becomes evident that even for the largest particle diameter in this range the flow regime does not always reside in the fully developed turbulent zone. This makes it necessary to extend the treatment of equations approximately to include data in this regime also, although the experimental data here are not comparatively plentiful. It becomes necessary, then, to extrapolate available ϵ_{mf} data beyond a particle diameter of 0.02 in. Inspection of the plot of the minimum fluidization voidage against particle diameter for various irregular shapes reveals that at about a particle diameter of 0.02 in., ϵ_{mf} tends to become independent of particle diameter and attain a constant value. These steady state values seem to be related to shape factor. A linear dependence is evident from inspection of Figure 4, where the extrapolated steady values of ϵ_{mf} for various materials are plotted against shape factor. The extended value of ϵ_{mf} for a bed of uniform spherical particles comes to 0.348. This value may be compared with 0.34 for random packing of uniform spheres (*10c*) for a fixed bed of large tube/particle diameter ratio (no wall effect) when one considers the fact

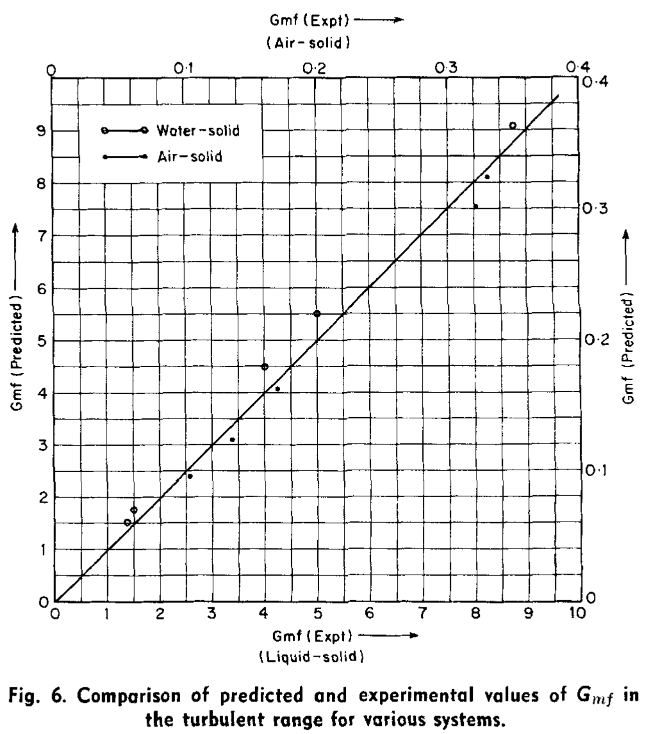

Fig. 6. Comparison of predicted and experimental values of G_{mf} in the turbulent range for various systems.

that ϵ_{mf} values are always slightly higher than those for stabilized fixed beds. Therefore the value assumed for ϵ_{mf} equal to 0.35, on which Equation (6) has been derived, appears to be justified. Based on the above considerations it is reasonable to relate the minimum fluidization voidage for a bed of particles larger than 0.02 in. to the shape factor below:

$$\epsilon_{mf} = (0.768 - 0.42\,\phi_s) \qquad (10)$$

This relationship may be introduced in Equation (5) to provide an expression for the prediction of G_{mf} for irregular particles larger than 0.02 in. This may be given as

$$G_{mf} = \frac{42.9\,\mu}{D_p} \cdot f_o(\phi_s)\left[(1 + 0.0056\,f_1(\phi_s) \cdot N_{Re_t})^{1/2} - 1\right] \qquad (11)$$

where

$$f_o(\phi_s) = \frac{(0.232 + 0.42\,\phi_s)}{\phi_s}$$

$$f_1(\phi_s) = \frac{(0.768 - 0.42\,\phi_s)^3}{(0.232 + 0.42\,\phi_s)^2} \cdot \phi_s^3$$

In order to facilitate computation of $f_o(\phi_s)$ and $f_1(\phi_s)$, Figure 5 has been prepared, where these functions have been plotted against shape factor.

DISCUSSION

The validity of Equation (6) for predicting G_{mf} is directly linked to the justification for assuming a value of 0.35 for the minimum fluidization voidage for a bed of uniform spheres that is independent of particle diameter. While it is known that for such a bed ϵ_{mf} is independent of particle diameter, its exact magnitude can be obtained either on the basis of experimental data or extrapolation procedure on data relating to other systems. Here the latter procedure has been adopted, and the extrapolated value of 0.348 checks well with the value of 0.34 reported for a fixed bed (*10c*) possessing no wall effect. The final test of the equation evidently lies in its ability to predict reliable values for minimum fluidization velocity.

The advantage of Equations (6), (9), and (11) over those that are in current use lies in the fact that the above equations are applicable to any fluid-solid system and irrespective of the operating flow regime, whether streamline or turbulent. The only disadvantage, perhaps not serious, may be the necessity to have prior values of ϕ_s, if Equation (11) is to be employed. In order to compare the accuracy of estimated values of G_{mf} from Equation (9) for shapes other than spheres, this equation has been plotted along with other equations in current use for the system glass beads-air. This appears in Figure 1, and it is evident that Equation (9) predicts values with a reasonable degree of accuracy comparable to that of Leva. The merit of Equation (6) for spheres is seen when it is applied to turbulent conditions. Experimental data in this flow regime are not as plentiful as they are in the streamline zone. However, about ten values are selected from the investigation of Wilhelm and Kwauk (*15*), and these are compared with those estimated from Equation (6). The concordance is excellent as is seen from Figure 6. Included in this plot are two values taken from the investigation of Baerg (2), and the estimated values are obtained through the application of Equation (11), taking ϕ_s equal to 0.86. Here too the agreement is good (Table 1).

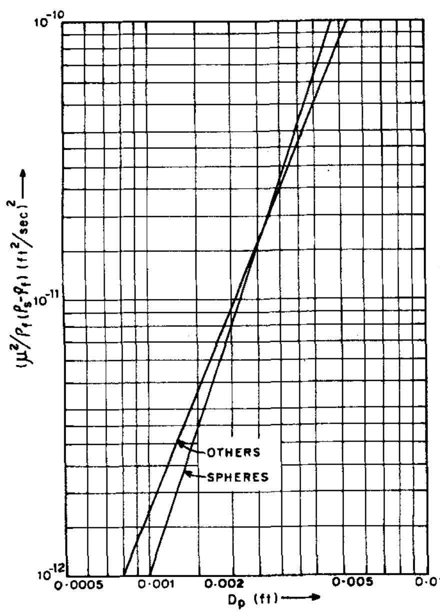

Fig. 7. Limiting values of particle diameters for convergence of Equations (6) and (9).

TABLE 1. COMPARISON OF EXPERIMENTAL AND PREDICTED VALUES OF G_{mf} IN THE TURBULENT REGION

System	D_p(ft.)	G_{mf} Exptl.	G_{mf} Predicted	N_{Re}	Author
Water-lead shots	0.0042	8.10	8.25	50	Wilhelm and Kwauk (*15*)
Water-glass beads	0.0171	8.05	7.80	222	Wilhelm and Kwauk (*15*)
Water-Socony beads	0.0145	4.26	4.07	92.3	Wilhelm and Kwauk (*15*)
Water-Socony beads	0.0107	3.40	3.10	55.1	Wilhelm and Kwauk (*15*)
Water-sea sand	0.0033	0.098	0.103	8.90	Wilhelm and Kwauk (*15*)
Air-glass beads	0.0171	0.350	0.363	495	Wilhelm and Kwauk (*15*)
Air-Socony beads	0.0145	0.200	0.220	238	Wilhelm and Kwauk (*15*)
Air-Socony beads	0.0107	0.160	0.180	130	Wilhelm and Kwauk (*15*)
Air-round sand	0.0029	0.060	0.072	14	Baerg (2)
Air-jagged silica	0.0024	0.055	0.061	10.9	Baerg (2)

The use of Equation (6) or (9) for predicting G_{mf} for fine low-density materials would need the expansion of the functions: $(1 + 0.00057\,N_{Re_t})^{1/2}$ or $(1 + 2.12 \times 10^{-5} \cdot D_p^{-0.55} \cdot D_p\,u_t \cdot \rho/u)^{1/2}$ in a binomial series. The accuracy involved in this approximation procedure depends on the number of terms included in the expansion. The condition for the convergence of the series is stated below:

$$\text{Spheres:} \quad D_p < 10\left[\frac{\mu^2}{\rho_f(\rho_s - \rho_f)}\right]^{0.33} \quad \text{for Equation (6)}$$

$$\text{Others:} \quad D_p < 63.8\left[\frac{\mu^2}{\rho_f(\rho_s - \rho_f)}\right]^{0.41} \quad \text{for Equation (9)} \qquad (12)$$

In Figure 7 the above equations have been plotted for easy evaluation of limiting particle diameter. It is to be borne in mind that the Reynolds number appearing in all the equations is based on the ultimate settling velocity of the particle assumed to be spherical, estimated in the streamline zone even if the magnitude of the Reynolds number indicates conditions otherwise. Accordingly this number may be defined as

$$N_{Re_t} = [D_p^3 \cdot \rho_f \cdot (\rho_s - \rho_f)\,g_c/18\,\mu^2]$$

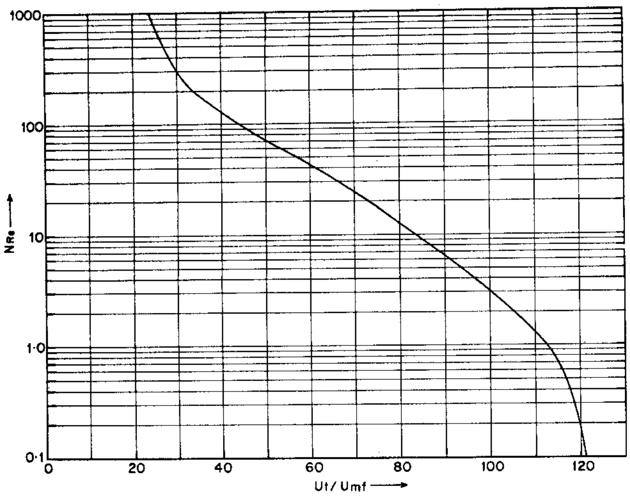

Fig. 8. Dependence of (Ut/U_{mf}) on Reynolds number for uniform spheres.

A few empirical equations have been proposed (*1, 14, 13*) for relating the ratio (u_t/u_{mf}) to the system variables. Brotz (*4*) proposed a value of 66 for this ratio, but this could only be an average value since the ratio is strongly dependent on the system properties. For a bed of uniform spheres, this velocity ratio may be estimated with precision by applying Equation (6) and noting that $u_t/u_{mf} = C_k \cdot N_{(Re)t}/(D_p \cdot G_{mf}/\mu)$. Accordingly

$$(u_t/u_{mf}) = \frac{C_k \cdot N_{Re_t}}{27.83[(1 + 0.00057\, N_{Re_t})^{1/2} - 1]} \quad (13)$$

In the above equation, C_k is the Stairmand correction factor (*12*) which is used to allow for the deviation from Stokes law on which N_{Re_t} is based when the system passes into the turbulent or even transition region. Figure 8 indicates the variation of the velocity ratio with the particle Reynolds number based on settling velocity in the streamline region. The variation in the magnitude of this ratio is from 120 to 25 corresponding to a range of Reynolds number 0.1 to 1,000, providing a firm basis for the average value proposed by Brotz. By the application of dimensional analysis Richardson and Zaki (*13*) concluded that the functional dependence anticipated for the velocity ratio (u_{mf}/u_t) may be stated as

$$(u_{mf}/u_t) = f'\,(D_p \mathrm{u} \cdot \rho_f/\mu,\, D_p/D_t,\, \epsilon) \quad (14)$$

It is interesting to compare the dependence of G_{mf} on ϵ_{mf} with the equation proposed by Steinour (*14*) who related the ratio of hindered settling velocity as follows:

$$(u_h/u_t) = \frac{\epsilon^3}{(1-\epsilon)} \cdot f(\epsilon) \quad (15)$$

If it is agreed that the environmental conditions prevailing in a bed at the point of incipient fluidization are somewhat similar to those during the hindered settling of the material (*1*), then the ratio (u_{mf}/u_h) may be assumed to be a function of bed voidage alone. This will then yield a correlation for the ratio (u_{mf}/u_t) similar to Equation (15). The exact form of the expression may be obtained if the function inside the square bracket in the right-hand side of Equation (5) is expanded in a binomial series:

$$G_{mf} = \frac{0.120\, \rho_f \cdot u_t \cdot \phi_s^2\, \epsilon_{mf}^3}{(1-\epsilon_{mf})} \left[1 - 0.0014 \frac{\epsilon_{mf}^3 \cdot \phi_s^3}{(1-\epsilon_{mf})^2} N_{Re_t} + \ldots\right] \quad (16)$$

When one notes that $G_{mf} = u_{mf} \cdot \rho_f$, Equation (16) becomes

$$(u_{mf}/u_t) = \frac{\epsilon_{mf}^3}{(1-\epsilon_{mf})} \cdot 0.120\, \phi_s^2 \left[1 - 0.0014 \frac{\epsilon_{mf}^3 \cdot \phi_s^3}{(1-\epsilon_{mf})^2} N_{Re_t} + \ldots\ldots\right] = \frac{\epsilon_{mf}^3}{(1-\epsilon_{mf})} \cdot f''\,(\epsilon_{mf} \ldots) \quad (16a)$$

The striking resemblance of Equation (16*a*) to Equations (14) and (15) confirms the belief that the process of expansion of a fluidized bed subsequent to the point of incipient fluidization is almost the reverse of that of hindered settling of the material.

NOTATION

C_k = Stairmand correction factor
D_p = particle diameter, ft.
D_t = tube diameter, ft.
f_0, f_1, f, f', f'' = functions
G = fluid mass velocity, lb./sq. ft. sec.
g_c = conversion factor, 32.2
L = bed height, ft.
N_{Re} = Reynolds number, $D_p \cdot u \cdot \rho/u$
u = fluid velocity, ft./sec.

Subscripts

f = fluid
h = hindered settling condition
mf = minimum fluidization
s = solid
t = terminal conditions

Greek Letters

ϵ = bed voidage
ϕ = shape factor
μ = viscosity
ρ = density
ΔP = pressure drop

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